Abstract: This paper asks what might be learned if we view undergraduate mathematics classes as sites of pedagogical preparation for teachers. Six courses in the same department were observed for a semester and media data (syllabi, textbooks, etc.) were collected. This paper discusses the striking differences in pedagogy across the courses for the purpose of highlighting possibility. In particular it examines the perceived needs of students related to proving (purposes) and the activities students are asked to engage in as participation in the course. Analysis suggests that students can witness a range of pedagogies to inform their teaching and poses the question: how can mathematics and education departments collaborate to take advantage of these opportunities?

Proofs, Purposes and Participation in Undergraduate Mathematics

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Introduction

This paper examines what aspects of the students in six upper level undergraduate math courses were supposed to change as a result of coursework related to proof (purposes) and roles students played in the courses (participation). From a broad view five of the six courses were lecture or direct teaching, but on closer inspection there were striking differences in instructor practice.

Although the experiences of all students are important, those pedagogic experiences of become extremely important in the cases of preservice secondary mathematics teachers—who are often math majors as well. It is hardly controversial to think about undergraduate mathematics classes as sites of teacher (content) preparation. This paper asks what might be learned if we view undergraduate mathematics classes as sites of pedagogical preparation for teachers.

Professors have the potential to provide powerful models of teaching although they may not explicitly teach pedagogy (Schoenfeld, 1989). Of the many aspects of pedagogy that might be examined, this paper focuses on the purposes of proof related coursework in six upper-level mathematics and the things students were asked to do as part of the respective course.

Literature

A common stereotype in popular culture is that mathematicians teach undergraduate classes using a mostly lecture or direct instruction approach (Picker & Berry, 2000). Reviewing many publications related to undergraduate mathematics (e.g., MAA, 1983; NAS, 1968; Rishel, 2000; Schoenfeld, 1990; Steen, 1989) suggests that the authors also assume this to be the case about their mathematician readership. By encouraging these professors to try new methods they are tacitly acknowledging a presumed similarity in their teaching—lecture. These assumptions might be accurate, but little research has been done on the nature of teaching in these settings (Robert & Speer, 2001). This work seeks to add to what is known about undergraduate mathematics teaching and also relate it to teacher education.

Theoretical Perspective, Methods, & Data

This paper draws on Lortie's (1975) widely cited theory of the Apprenticeship of Observation to suggest that what students witness has the potential to inform their future teaching, but it departs slightly by taking the theory, which has most often been found relevant to teachers' past experiences in their own K-12 coursework (e.g. Bird, 1991; Holt-Reynolds, Fall 1991), into the undergraduate setting.

Because Lortie's theory is concerned with the acts of teaching that students witness this project takes a phenomenological approach. It is the phenomena, after all, that comprise students' experience. How the instructor intends a teaching episode may tell us something important about teaching, but it does not change what the students could see and hear. This study observed six courses (two Linear Algebras, Real Analysis, Modern Geometry, Complex Analysis & Discrete Mathematics) approximately once every other week during a fall semester and focused on the nature of the activities during class time, the actions of the professor and students, the talk and questions, and the physical environment of the class. In addition to the detailed ethnographic field notes taken from each observation, the researcher gathered syllabi, handouts, textbooks, websites, and other forms of media made available to the students. The project takes a critical approach to analysis in that it looks for difference through nuance rather than similarity across a department for the purpose of highlighting the variety of pedagogic models available for preservice teachers.

The Courses

Across the courses students were positioned as needing a variety of changes in their relationships to proof and included Writing a Proof (both section of LA), Confronting the Role of Intuition (RA), Conjecturing (MG), Using and Distinguishing between Informal and Formal Approaches (CA), and Valuing Proof's Role in Mathematics (DM). Activities included but were not limited to sitting, taking notes, listening, watching, working in pairs, teaching short lessons, using technology, and challenging the instructor. For the purposes of this short paper, this section will focus on the Real Analysis and Modern Geometry courses. A primary aspect of the student in need of change in these courses were *Confronting the Role of Intuition* and *Conjecturing*, respectively.

In the Real Analysis course the professor often referred to student's intuition as a source of strength and as a way to develop and/or understand a proof, but cautioned them against relying on it. He structured activities so that students discussed their initial ideas in small groups/pairs before guiding them through proofs that either validated their early assumptions or showed how they were misleading. He often posed examples of tricky sequences to highlight the ways that intuition can be misleading but never asked them to abandon their intuitions altogether. During class time students were expected to take notes, listen, ask questions, brainstorm in small groups, and offer ideas for public consideration. The course attempted to help students confront their intuitions so as to become better at proving.

The Modern Geometry course took conjecturing as a main activity. Nearly every day the students were given some geometric scenario and were asked not to prove a given theorem, but instead to conjecture possible relationships. This portion of the class time was usually conducted in pairs. Later the class came together and shared their conjectures and jointly developed proofs for ones usually selected by the instructor. To do this they experimented through the use of technology, thought privately, debated with one another and even challenged the professor.

Discussion

Students in the Real Analysis course were positioned as between two epistemic worlds—their informal knowledge and the knowledge of the field. Preservice teachers could take away a lesson that the role of mathematics teaching is to guide students from their incomplete but not entirely absent knowledge towards established knowledge. In the Modern Geometry course students created and proved their own mathematics, although much of what they did had already been conjectured and proven (much) earlier in history. Students were positioned as capable thinkers in their own right. Preservice teaches could learn the pedagogy of letting students create their 'own' mathematical ideas. These are but two examples of the different 'lessons' available to preservice teachers.

Both Cuoco (2003) and Wu (1999) encourage other mathematics professors to think carefully about their roles as teachers and the lessons about teaching and mathematics these roles can impart on would-be teachers. Broadening our understanding of these undergraduate environments offers new ways to think about the role of undergraduate mathematics in preservice teacher preparation and the possible collaborations between mathematics and teacher education departments.

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